Month of Math

PROBLEMS AND SOLUTIONS 2025

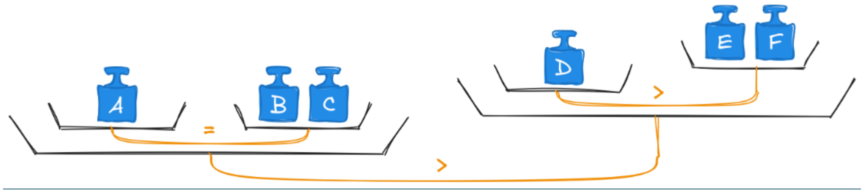
The Joy of Thinking Foundation

Contents

Problems

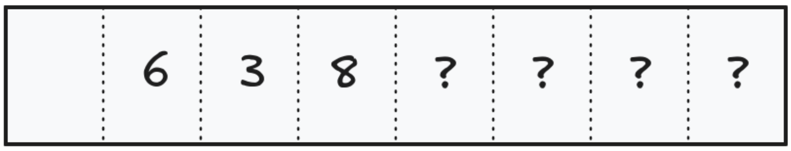
Scales on scales

On the big balance below, we put two identical balances and weights in their pans. The six weights weigh 1, 2, 3, 4, 5, and 6 grams. Can you figure out which weight was placed in which pan? (solution)



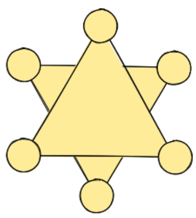
Paper folding

Fold the sheet below along the dotted lines so that they form a stack, then write in each section which layer it ends up in when counted from the top (it's okay if the number appears on the reverse side of the paper). How should the paper be folded to get these numbers? (solution)



Sheriff badge

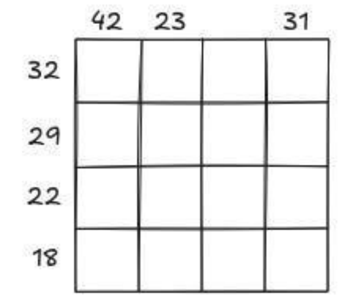
First, place two congruent equilateral triangles on top of each other so that they overlap at a regular hexagon. Then attach six discs so that the centres of the discs are at the vertices of the triangles. This gives you a sheriff badge:



The area of a triangle is 36 units, and the area of a disc is 4 units. What is the area of the badge? (solution)

Coins from Madagascar

In Madagascar, coins of 1, 2, 5, 10, 20, and 50 ariary are in circulation. Place one coin in each cell of the grid so that the numbers in the rows and columns indicate the total value of the coins in that row or column. (solution)



Power outage

When Lisa left for school in the morning, all three of her clocks showed the correct time. But during the day there was a power outage, so when she got home in the afternoon, the wall clock read a quarter after four, the clock on her radio showed 16:25 and the digital clock on her desk 14:40.

The wall clock is not affected by power outages, but the digital clock and the radio are switched off when the power goes out. When the power comes back on, the clock on the radio restarts from 12:00, while the digital clock continues from the time when it stopped at the start of the blackout.

When did the blackout at Lisa's house start and end? (solution)

Graphics of Mond Ryan

Mond Ryan, a renowned graphic artist, is experimenting with drawings created with broken lines. He is currently in his six-segment phase: creating drawings with broken lines consisting of six straight segments (five breaks in the line). He has noticed that the more triangles he has in a picture, the higher the price he can sell it for. Only non-overlapping triangles count -- that is, triangles which don't have another line inside them. He has managed to create drawings with five triangles, but he won't stop there.

Try to create as many triangles as you can with a broken line consisting of six segments to inspire the master! (solution)

Heads of dragons

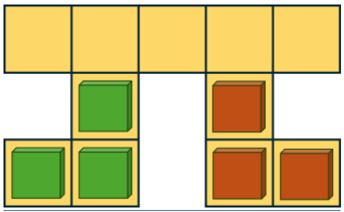
Shesha the dragon invites his many-headed dragon friends to a council meeting in the Himalayas. Including Shesha, there are five participants in the discussion. According to tradition, they sit in a circle and examine each other's heads. Shesha speaks first, and then, moving to the right, they say the following in Sanskrit:

- The द्वौ neighbors sitting to my right have an average of 7 heads.
- The द्वौ neighbors sitting to my right have an average of 9 heads.
- The द्वौ neighbors sitting to my right have an average of 8 heads.
- The द्वौ neighbors sitting to my right have an average of 8 heads.
- The द्वौ neighbors sitting to my right have an average of 9 heads.

Unfortunately, the interpreter did not translate what द्वौ means. How many heads do the dragons have in total? (solution)

The π-shaped storage room

We're looking at the storage room from a top-down view. During the night shift, the boxes were placed in the wrong spots, so before anyone notices the mistake, we'd like to swap the green and brown boxes. We can slide the boxes on the floor to any reachable empty square. Only one box can occupy a square at a time, and we can't lift or carry them over other boxes manually—unfortunately, the forklift operator has the day off.



Can you rearrange the boxes? (You can even try it out online.) How many moves does it take? (solution)

Truth-tellers and liars in the Legion of Janus

In ancient Rome, the Legion of Janus consisted of 50 soldiers arranged in a 10×5 rectangle. Each soldier was either a truth-teller or a liar. Every one of them made the following statement:

"All of my comrades in my row and column are liars."

How many truth-tellers are there in the legion? (solution)

Candies for Christmas

For the school Christmas celebration, parents sent candy for the children in a festive bag. A total of 100 candies were placed in the bag, but they didn't tell the students how many different flavors they had bought. However, they did say that there was an equal number of each flavor. What is the minimum number of candies the students need to eat to be certain of how many different flavors of candy are in the bag? (solution)

Code of the suitcase lock

I bought an extra-secure suitcase with a lock that can only be opened by entering its 10-digit code. I can set each digit between 0 and 9 as I wish. I was very happy with the purchase, but I soon realised that I will need to remember my extra-secure code without writing it down. Then I had a fantastic idea - I will set the code so that it reveals information about itself:

The first digit tells me how many 0s are in the code, the second tells me how many 1s are in the code, and so on, the 10th digit tells me how many 9s are in the code. I am sure I will not forget this. What is the code on my lock?

My friend Andi told me that she wouldn't want to figure out a code that long every time she is at the airport, but she liked my method of setting up the code. So, she wants to buy a lock for which she can set a code in the way described above, but she wants the code to be as short and as small as possible. What should her code be? (solution)

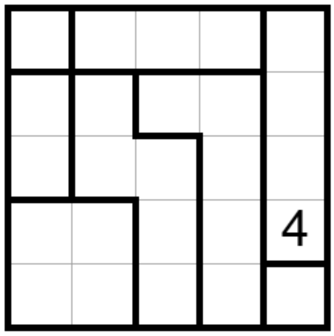
What is my number?

I picked a number between 1 and 10 (possibly including 1 or 10). You can choose any number, and I'll tell you if it's *close* to the number I picked. What does *close* mean? It means the two numbers are either the same or neighbors (that is, their difference is at most 1). Try to figure out the number I picked! Once you've found it out, see if you can do it in as few questions as possible!

You can also try it online. (solution)

Non-neighbouring numbers

Consider the grid below divided into regions of various shapes and sizes. Fill in the squares so that each region of size N contains each of the numbers from 1 to N exactly once. You may not write the same number in any two squares that touch, even diagonally. (solution)



On the shores of Themis

On one side of the remarkably straight River Themis live truth-tellers, and on the other side live liars. Professor Dialectos, the renowned ethnographer, during one of his journeys, spoke with six locals who live at the vertices of a regular hexagon. He recorded what they said, but unfortunately, he forgot which side hosts the truth-tellers and which the liars. Worse still, he can't find in his notes exactly where the straight river cuts through the hexagon.

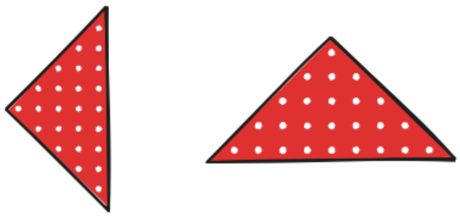
To preserve anonymity, he labeled the residents with the letters A, B, C, D, E, and F, who live in adjacent vertices in that order. They made the following statements:

- **A**: D and E do not live on the same side.
- **B**: I live on the same side as F.
- **C**: F is a liar.
- **D**: I live on the same side as B.

Help Professor Dialectos figure out who is telling the truth and who is lying! (solution)

Grandpa Tarkal's scarf

Grandpa Tarkal wants to sew polka-dotted silk scarves for his four granddaughters. He knows that the traditional scarf pattern is shaped like an isosceles right triangle, and each granddaughter needs a different size. However, upon returning home from the fabric store, he is shocked to realize that they misunderstood him and gave him one large quadrilateral piece of fabric. After a brief moment of thought, he slaps his forehead and realizes that he can cut the fabric in such a way that all four scarves can be made from it, without wasting any material!



What might the fabric Tarkal Grandpa brought home have looked like? (solution)

Better and better questions

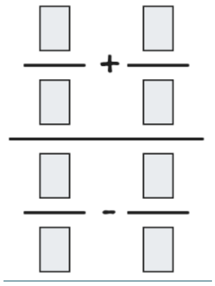
I picked a number between 1 and 14 (possibly including 1 or 14). In each round, you can guess one number. Starting from the second round, I'll tell you if your new guess is closer to the target number than your previous guess was. (I won't tell you if your guess was correct. You have to know for sure.) Try to find the number I picked! Once you've found it, try to do it in as few guesses as possible! You can also try it out online. (solution)

DIY custom picture frame

The company PictArt has just launched its latest product: a DIY custom picture frame. It's a square, wooden, open frame with 8 attachment points—one at each corner and one at the midpoint of each side. The package includes 4 elastic cords that can be stretched between these attachment points. Each attachment point can hold the end of only one cord, so all 8 points are used exactly once. Once the cords are in place, you can hang pictures at the intersection of the cords, using specially designed hooks. (If more than two cords intersect at a single point, unfortunately, you can still only hang one picture there.) How many pictures could this frame possibly hold? In other words, what's the maximum number of intersection points the cords can form? (solution)

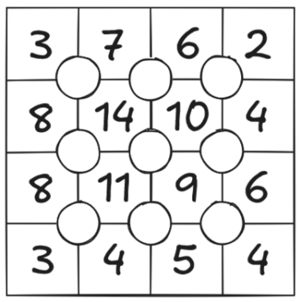
Fraction of fractions

Write the numbers 1, 2, 3, ..., 8 into the following expression. Find an arrangement that makes the value of the fraction as large as possible. Are you sure you have found the largest one? (solution)



Vault

There is a combination lock which is a 4 × 4 square, each cell of which starts with the number 0. There are some circular buttons on the lock, each corresponding to the 2 × 2 part around it. When a button is pressed, all numbers in its section increase by 1. We got ahold of the correct combination, depicted in the figure. The vault only unlocks if we manage to set the correct number in each of the 16 cells. (solution)

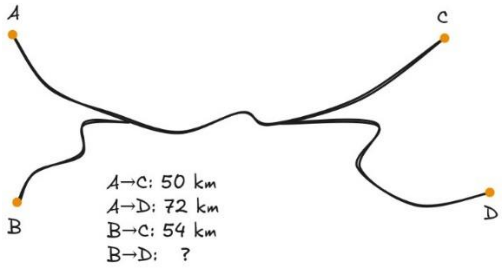


Wobbly Table

The table in this animation is a wobbly one (click on it to see its motion). The table has standard dimensions: it's 75 cm high with a 90×120 cm tabletop. Two legs are the correct length, while the other two are slightly shorter—each by less than 2 cm, though we don't know exactly by how much. We do know the shorter legs are of different lengths and sit opposite each other along a diagonal. We observe that the wobbling table has two stable positions. In each one, a leg is lifted off the ground. Do those lifted legs rise to the same height? If not, which one is higher? (solution)

Sketchy map

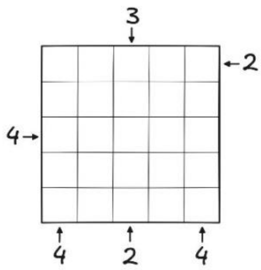
A very sketchy map (see below) has been found:



Can we determine the length of the route from B to D? (solution)

Housing complex

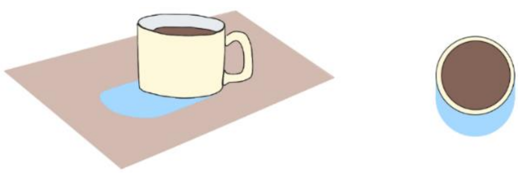
There are 25 buildings in a 5×5 grid in a housing complex. Each building has 1, 2, 3, 4 or 5 floors. In each row and each column, there is exactly one building of each height. A fan of brutalist architecture walked around the housing complex, looking straight across rows and down columns to see how many buildings were visible from that direction. A building is only visible if there are no taller buildings in front of it in that row or column. The diagram below shows their observations:



Write the number of floors for the building in each cell of the grid of the housing complex. (solution)

Morning Coffee on the Table

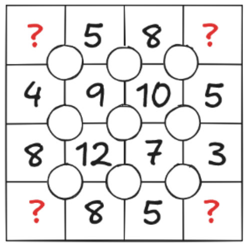
I noticed too late that the bottom of the coffee mug was wet, and I placed it on the table. Then I slid it 2 cm (in a straight line). This resulted in the water stain shown in the illustration:



What is the area of the water stain **outside** the mug if the mug's diameter is 8 cm? (solution)

Vault with 12 cells

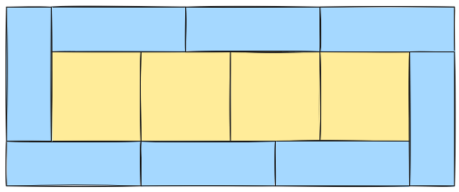
We’re faced with yet another combination lock made up of a 4 × 4 grid. The structure and operating mechanism of the lock are the same as those we’ve encountered before: pressing a circular button increases the value of each of the numbers in the surrounding 2×2 area by one. Initially, each cell starts with the number 0. However, to open the lock now, it’s sufficient to set the values of the 12 cells shown below correctly, regardless of the numbers in the corners marked with question marks.



At first glance, this might seem easier—after all, we only need to focus on 12 cells instead of 16. But is it really an advantage? How can we open the lock this way? (You can try it online, too.) (solution)

Mosaic floor

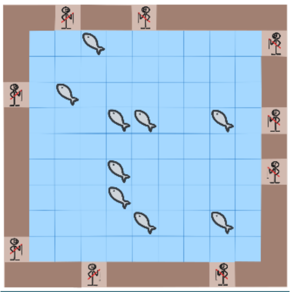
In Gorsium, a large room decorated with a mosaic floor was uncovered during excavations. The floor plan of the room is shown in the illustration.



The central yellow parts are squares, and the outer blue parts are identical rectangles. We know that the perimeter of the room is 42 meters. What is its area? (solution)

Fish in a lake

Some anglers are standing around a lake, and each of them has just hooked a fish. We know that everyone caught their fish using 9 units of fishing line, that is, each line passes through 9 tiles, excluding the tile of the angler but including that of the fish. Draw the paths of the fishing lines! (You can also do so online).



The lines can only pass between tiles with a common side, can not branch, and each tile can only have at most one line passing through. (solution)

Island of different tea types

Far, far away, on a perfectly circular island, world-famous teas are grown. The island has been divided in a special way so that there are two areas for cultivating two unique types of tea:

- The two tea fields have equal areas, each covering half of the island's total area.
- Each tea field's perimeter is the same as the perimeter of the island.
- The fields form contiguous areas for each type of tea.

Draw a division that satisfies these conditions!

There are many possible constructions — try to find one that's as simple and easy to draw as possible! (solution)

With friends like these...!

There is a group of 12 people consisting of truth-tellers and liars. We also know that everyone has exactly two friends within the group. (Friendships are mutual.) Everyone makes the following claim: “Both my friends are liars.” What is the maximum number of truth-tellers in the group? And what is the minimum number of truth-tellers? (solution)

A stream, rails, and a trail

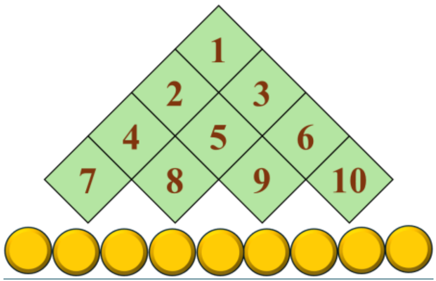
Next to a hiking trail meanders a stream and a railway line. At the beginning of the hike the stream is on our left side, and the rails are to the left of the stream. We crossed the stream eight times and the rails five times in total. Can it be determined in what order the stream, the railway, and the trail are at the end of the hike? (solution)

Let’s jump!

Draw a special board (see below), then place nine tokens on nine squares however you like (you can use buttons or coins)! The goal of the game is to remove all but one of the tokens from the board.

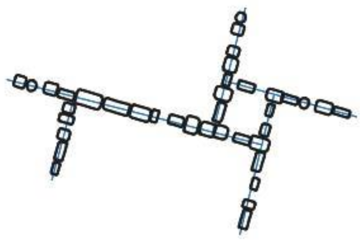
In one move, you can jump over an adjacent token with any of your tokens; the jumped-over token is then removed from the board. Only one token can occupy a square at any time! Adjacent means squares that share a side or a corner — so you can jump, for example, from 7 to 9 or 2. No other moves are allowed; every jump must result in removing a token.

If you can continue jumping with the same token onto another empty square following the rules, you may do so any number of times in a row. As long as you don't release the token, your jumps are considered part of a single step. Try to remove all the tokens but one using as few steps as possible! (solution)



Brickville

It has long been known that a castle once stood near the borders of Brickville, though now it is just a pile of stones. A surviving letter from a Walloon captain mentions that the building was a single-story rectangle divided into six square rooms. Recently, archaeological excavations uncovered parts of the foundation, matching the captain's description, as shown in the sketch attached to the excavation report. One room is fully outlined among the remains, with walls measuring 10 feet in length along their centerlines. Since the wall thickness is roughly the same everywhere, we will disregard it and represent the walls by their centerlines. Let's draw the full floor plan of the castle and determine its dimensions! (solution)



Island of different tea types 2

We're once again visiting a circular island where tea is grown, but this time there are four types of tea being cultivated. Here's what we know about the tea fields:

- The four tea fields have equal areas, each covering one quater of the island's total area.
- Each tea field's perimeter is the same as the perimeter of the island.
- The fields form contiguous areas for each type of tea.

The island, with its plantations, also offers a wonderful view — this unique layout even appears as the logo on the tea packaging! :)

Draw a division that satisfies the conditions above! There are many possible constructions — try to find one that's as simple and easy to draw as possible!

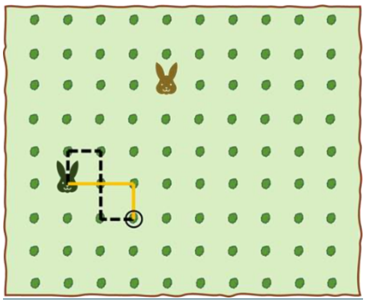
Can you also draw a division where the four tea fields are *not* all the same shape? (solution)

Non-trapezoid

Draw a quadrilateral with no parallel sides that can be divided into 4 congruent (identical) triangles! (solution)

Cabbage field

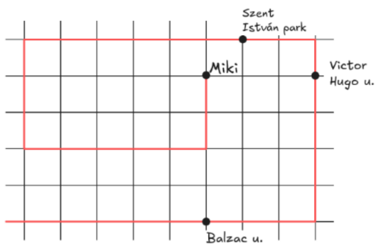
Two rabbits grow their cabbages together on a fenced-off rectangular plot. In the figure below the two rabbit pictures mark the corresponding rabbit holes. The cabbages are planted in a square grid pattern. The soil between them was ploughed loose, so the rabbits stick to traveling by hopping around, always jumping to one of the closest cabbages – in particular, they cannot jump diagonally. For example, the black dashed path is a valid 5-jump-long path to a ready-to-harvest cabbage, but this same spot can be reached in many different ways by the black rabbit, for example by using the yellow 3-jump-long path.



The cabbages are ready to harvest soon. Following their tradition, the rabbits donate those cabbages which are the same number of jumps away from the two rabbit holes to a noble cause. To each cabbage they count the least number of jumps. For example, the circled cabbage on the figure is 3 jumps away from the black rabbit's hole, and 5 jumps away from the brown rabbit's. How many cabbages are donated this year by the two rabbits? (solution)

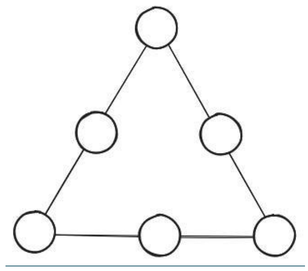
Trolleybus 76

The map below shows a section of trolleybus line 76. The black lines represent streets, while the red line marks the route of the trolleybus. Miki is currently at a stop, but the trolleybus left 5 minutes ago. However, he knows the route is winding, so he might be able to catch up at a later stop. If Miki hurries to either the Szent István Park or Victor Hugo Street stop, he will arrive at the same time as the trolleybus. Suppose Miki and the trolleybus both move at a constant speed (we ignore the time spent at stops). How long would Miki have to wait if he went to the Balzac Street stop? (solution)



Pebbles in a triangle

Place pebbles into the bowls arranged along the sides of the triangle. Use every possible number of pebbles from 1 to 6, and make sure that the total number of pebbles is the same along each side.



Submit the numbers in the order they appear around the perimeter of the triangle, starting from one of the vertices and following in some direction around the triangle, as a six-digit number. (solution)

Dystopia

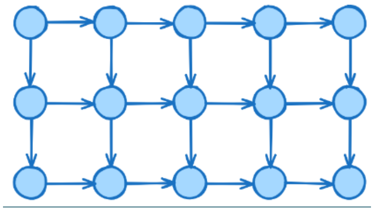
On a server, we are chatting with 12 artificial intelligences and one human, and our goal is to figure out which one is the human. It would be impolite to directly ask someone if they are human, so we instead ask the following type of question: we ask A if B is an artificial intelligence.

Everyone is truthful. The possible answers are either "yes" or "I don't know." The AIs are sometimes uncertain whether someone is an AI or not, but the human knows for sure that the others are not human.

Your task is to find a strategy that identifies the human with the fewest number of questions possible. (solution)

Contamination

The diagram below shows a map of 15 pools at an aquatic center. The arrows indicate the direction in which the water flows from one pool to another.



We know that one of the pools has been damaged and is releasing contamination into the water.

You have a device that can detect whether the water in a pool is contaminated. Every pool downstream of a contaminated pool will also be contaminated.

Assuming the worst-case scenario, what is the minimum number of tests needed to guarantee finding the source of the contamination? (solution)

Picture book

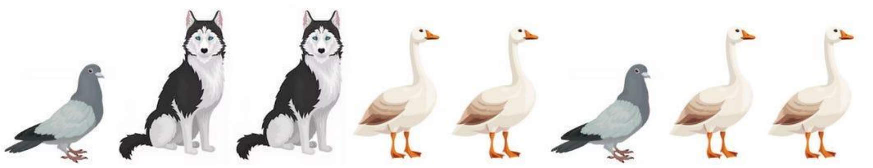
A small child is looking through their favorite picture book, which features three types of animals: a pigeon, a goose, and a dog. Each page contains exactly one animal and the child flips through the book from beginning to end looking at the animals page by page.

Whenever they see an animal and realize that they've now seen this animal more times than any of the others, they imitate its sound. In all other cases, they stay silent. As a special rule, they also imitate the sound of the very first animal.

While they're flipping through the book, we hear the following:

coo-coo, coo-coo, honk-honk, woof-woof, coo-coo, coo-coo, woof-woof, woof-woof, honk-honk.

How many geese did the child see?



In the above order, we would hear the following sounds: coo-coo (from the first pigeon), woof-woof (from the second dog), honk-honk (from the third goose), honk-honk (from the fourth goose). (solution)

Soccer field

A soccer team of 11 players is at a training session. In one of the exercises everyone has a ball, and their task is to – at precisely the same moment – pass the ball to the teammate closest to them. (Assume that any two players are at a distinct distance from each other).

We would like as few players as possible to have balls after the passes. What is the minimum number of players who can have balls after the passes? You can even try it out online. (solution)

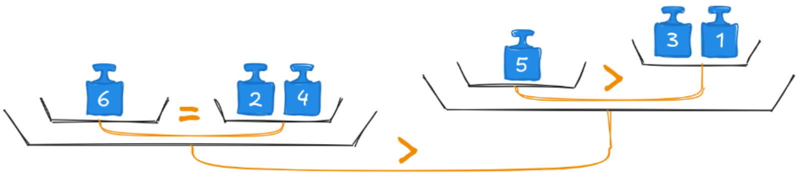
Solutions

Scales on scales

The total mass of the six weights is 21 grams, so the heavier pan (left) should have at least 11 grams of weight. The balance on the left side is in equilibrium, which implies that the total mass is an even number, and together with the previous observation, this means that the total mass on the left large pan (A+B+C) is at least 12 grams.

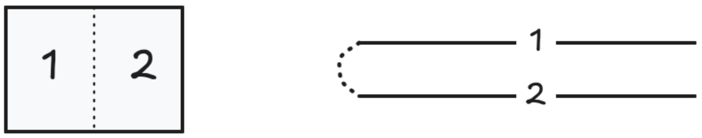
The only way to do this is to put a weight of 6 grams at position A. Likewise, B and C must be either the 5 and 1 gram weights, or the 4 and 2 gram weights. If we put the 5 gram weight in place of B or C, then 2, 3, and 4 gram weights would be placed in the right large pan. However, we cannot place them in the desired way, since 4 < 2 + 3.

So we need to check if the 2 and 4 gram weights can be placed at position B and C. This does indeed lead to a good solution, since 6 = 4+2, and 5 > 1+3 (and 6 + 4 + 2 > 5 + 3 + 1). We also see that we have found the only possible solution, the product of the masses of the three weights on the right side is 15=5·1·3. (problem)

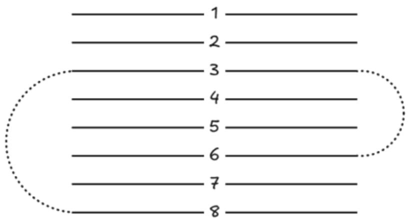


Paper folding

In the solution, we reverse the problem. We start with a stack of paper with the top sheet labeled with a 1, the bottom with an 8, and those in between labeled sequentially. We then imagine gluing together edges to make a continuous sheet that, when unfolded, will reveal the sequence given to us in the problem (we allow that some of the numbers might not be on the same side of the unfolded continuous sheet). In the diagram below, the stack of paper is viewed from the side. A dashed arc indicates two sheets are glued together. We will only glue together two edges that are on the same side of the stack and we will never rearrange the stack. We also will not allow dashed arcs to intersect. In this way, gluing together two edges is equivalent to making a fold. For example, if there were only one fold on the sheet, we would represent it like this:



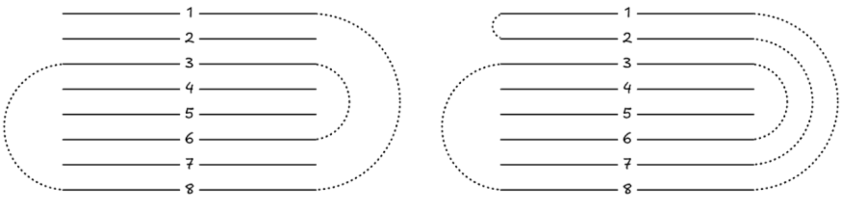
At the beginning, our diagram looks like this:



In the solution, we will use the fact that the connecting arcs cannot intersect, as they represent the arcs of the folds. We will also pay attention to where each layer can be positioned on the unfolded sheet.

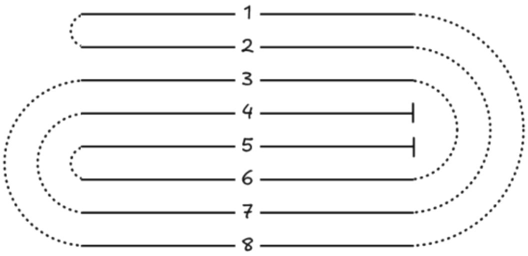
The loose end of 8 cannot be connected to 7, as that would make it impossible to connect sections 1 and 2 with the rest. If 8 were connected to 2, then the left end of 2 could only be connected to 1. The other end of section 1 couldn't connect to anything else, meaning 8 would be two sections away from the edge of the sheet, which contradicts the layout of the unfolded diagram.

Therefore, 8 can only be connected to 1 (left-hand diagram). Based on this, we can already draw a few more arcs (right-hand diagram).



If we were to connect 6 with 4, we could only continue with 5, which would have to be at the edge of the sheet. However, in that case, there would be at least two sections on either side of section 6 on the unfolded paper, which would be a contradiction. If 6 were connected to 7, it would form a loop: 6-7-2-1-8-3-6.

Therefore, 6 can only be connected to 5 The other side of 5 is one end of the continuous sheet. Then, the remaining loose end of 7 must be connected to 4, which is the other end of the paper.

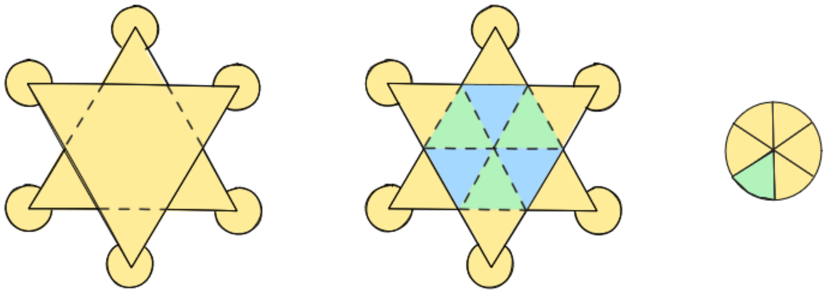


When the sheet is unfolded, the order of the numbers is 5-6-3-8-1-2-7-4, so the answer is 1274.

(problem)

Sheriff badge

Let’s take a look at the badge from back:



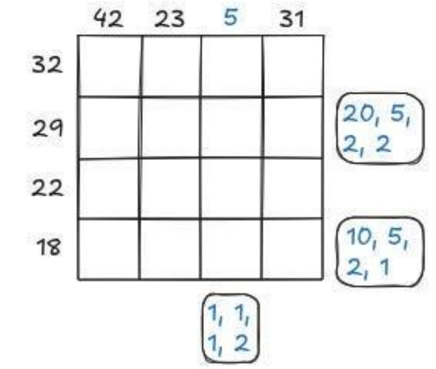
Let’s connect the vertices of the regular hexagon as shown in the middle diagram. This forms equilateral triangles whose sides are all equal. Each large triangle is made up of nine small triangles, meaning each small triangle has an area of 36/9=4 units. Therefore, the star has a total area of 36 + 3×4 = 48 units.

The badge is completed by adding the disks. In the diagram on the right side, we can see that the triangles cut out a 60-degree sector from each disk, making up exactly one-sixth of the disk's area. The remaining 5/6 of each disk adds to the total area of the badge, so the disks contribute 4*5=20 additional units. Thus, the total area is 48+20=68 units. (problem)

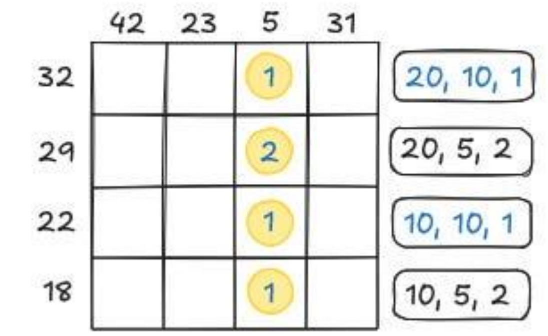
Coins from Madagascar

Since we know the total value of the coins in all four rows, we can determine the total value of all the coins in the grid: 32 + 29 + 22 + 18 = 101. From this, we can determine the sum of the coins in the third column: 101 - 42 - 23 - 31 = 5.

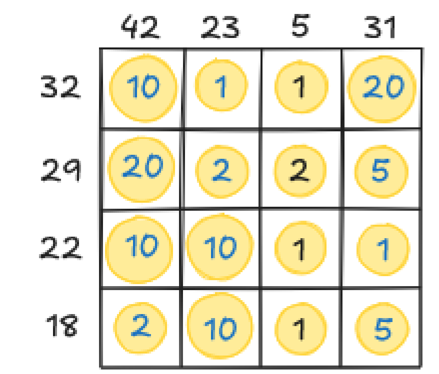
For some rows and columns, we can determine the exact denominations that make up the given total. These are written next to the rows and columns:



From this, we see that the third cell in the second row can only be a 2. This means that all other cells in the third column must contain a 1. With this information, we can also determine the denominations for the first and third rows.

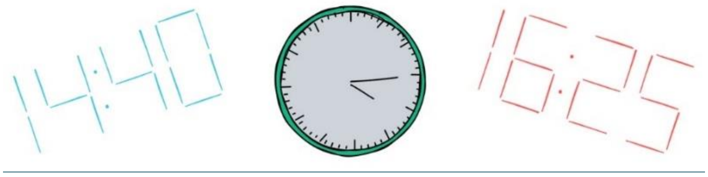


The numbers in the fourth column must sum to 31, which can be broken down in two ways. If we split it as 10 + 10 + 10 + 1, we wouldn't be able to place a valid coin in the second row. Therefore, the only valid breakdown is 20 + 5 + 5 + 1, meaning the coins must be placed in the order 20, 5, 1, 5. For the 23 in the second column, among the remaining possibilities, only 10 + 10 + 2 + 1 works, arranged as 1, 2, 10, 10. From here, the first column can also be uniquely determined (the answer is 10, 2, 1, 5). (problem)



Power outage

Let’s look more closely at how each of the clocks behaves.



The wall clock is not affected by the power going out, so it shows the correct time: Lisa got home at 16:15.

The clock on the radio restarts at 12:00 when the power comes back on. As it is currently showing 16:25, it has been 4 hours and 25 minutes since the power came back on.

The digital clock now reads 14:40 and it's been 4 hours and 25 minutes since the power came back on, so it must have read 10:15 when it restarted. Since this clock resumed from the time the blackout started, the power must have gone out at 10:15.

Now let’s see how long the blackout lasted. Once again, the digital clock helps us. Because it resumes from the same place it stops, the difference between the time it shows and the true time tells us how long the power was out. The digital clock now reads 14:40, but the real time is 16:15. That’s a difference of 1 hour and 35 minutes, which is exactly how long the blackout was.

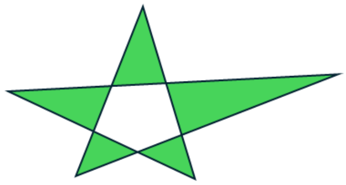
Since the blackout started at 10:15 and lasted 1 hour and 35 minutes, the blackout lasted from 10:15 to 11:50. (problem)

Graphics of Mond Ryan

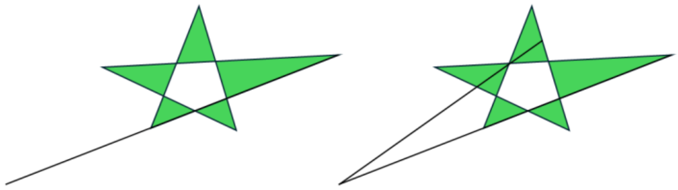
Seven triangles can be done. Pause here if you still want to try to find such a drawing.

This puzzle was inspired by a question of Kobon Fujimara, known as the Kobon triangle problem. He asked: how many non-overlapping triangles of this type can be created by n lines? In the Kobon triangle problem, the lines do not have to be connected segments as in our puzzle. Since then, many people have worked on this question, but no formula for n lines has been found. Now, let’s focus on our specific problem:

You may want to start experimenting with fewer sections. With four sections, things don't look very exciting, so let's see what you can get out of five! With a bit of trial and error, we can easily get to the following star, with which we have already achieved five triangles:



With six segments, the situation becomes much more interesting. Let's see if we can extend the star drawing from five to six segments and introduce some more triangles. If we start by extending the last segment of the star, we can finish it in several ways to get two more triangles. One possible ending (in the figure on the right):



Of course, there may be different approaches to drawing with six segments, and it will take some convincing to see that more than seven triangles is impossible. Experimentation itself is interesting, as we gain experience, create interesting drawings, and ask new interesting questions along the way. For example, with six connected segments, how many triangles can I create without forming any polygons other than triangles, like the white pentagon in the example above? We can also get a feel for what makes a 'good' drawing. For example, it matters exactly how you draw lines through existing triangles, as you don't always get more triangles as a result...

Under the Kobon Fujimara condition, no more than seven triangles can be formed with six (complete) lines, so seven is indeed the answer. Seven lines can create eleven Kobon triangles. With our condition, i.e. with connecting the segments, our team has managed to draw ten triangles. Is it still possible to make eleven? (problem)

Heads of dragons

If द्वौ = 1, the task would be easy because each dragon would directly reveal the number of heads their right neighbor has.

However, we can still solve the problem even if द्वौ is not 1. When a dragon is included in an average, its heads are counted with a weight of 1 / द्वौ, since द्वौ dragons were averaged. Each dragon appears in the averages of the द्वौ dragons sitting to its left, meaning it is counted exactly द्वौ times.

This means that in the sum of all averages, each dragon's heads are counted exactly once, regardless of the value of द्वौ. Thus, the sum of the averages: 7 + 9 + 8 + 8 + 9 = 41 directly gives the total number of heads. (problem)

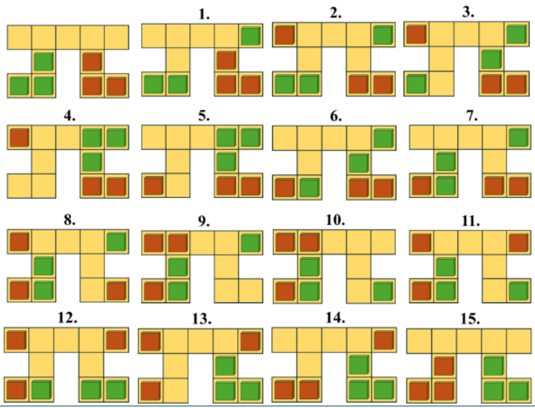
(द्वौ actually means "two" in Sanskrit. In this case, the individual head counts can be determined as: 7, 7, 11, 5, 11.)

The π-shaped storage room

On your first attempt, it might easily feel like the whole thing jams up at the moment you start — maybe it's not even possible to rearrange the boxes. But with a bit of experimenting, you'll get the hang of it and become a more skilled mover.

We believe that at least 15 moves are necessary — here's a 15-step solution a step-by-step image:

Step by step:



(problem)

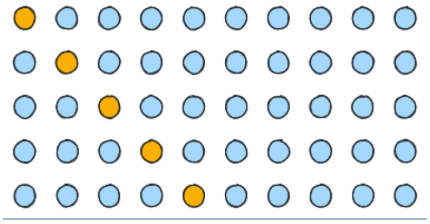
Truth-tellers and liars in the Legion of Janus

There can be at most one truth-teller in each row, because if there were more, they wouldn't all be telling the truth. So there can be at most five truth-tellers.

If a liar makes such a statement, then there must be at least one truth-teller in their row or column. Thinking this through, we also see that every row must contain a truth-teller. If there were a row of only liars, then at the intersection of that row and a column that also contained only liars, we would have a soldier who is a liar and could not have made the the given statement (because it would have been true).

There must be at least one such "all-liar" column, since there are 10 columns and at most 5 truth-tellers.

Therefore, the number of truth-tellers must be exactly five — no more, no less. We can even construct an example, marking the truth-tellers in orange. We leave it to the reader to verify that in this configuration, every soldier could indeed have made the statement given in the problem. (problem)



Candies for Christmas

It's useful to think about the problem this way: if we wanted to hide the number of different flavors for as long as possible, how should we distribute the candies to the children?

Let's start with a simpler example: say there are only 12 candies. Suppose the first 5 candies drawn have the following flavors: 1, 1, 1, 2, 3. (For simplicity, we'll number the flavors.) Can we already determine how many different flavors there are? Unfortunately, no — there might be just these 3 flavors, with 4 candies each, but it's also possible there are 4 flavors, each with 3 candies.

What if the first 5 candies all had different flavors? Even then, we still can't be sure. There could be 6 different flavors, each with 2 candies, or even 12 different flavors altogether. However, there definitely can't be just 5 flavors, because 12 isn't divisible by 5 — meaning there wouldn't be an equal number of each.

Now, let's return to the original problem. Imagine that the children have already eaten 80 candies, and it's still unclear how many flavors there are. If they had received exactly 20 candies from each of 4 flavors, then two possibilities remain:

1. There might be a total of 4 flavors, with 25 candies of each.
2. Or there might be 5 flavors, and the bag still contains exactly 20 candies of a fifth flavor that hasn't been tasted yet.

Now comes the more difficult part of the solution — showing that after eating 81 candies, it's guaranteed that we can determine how many different flavors there are.

Let's consider when two different distributions of 100 candies might still be indistinguishable. Suppose one case has x flavors, and another has y flavors (with $x < y$). Since each flavor is represented equally, x and y must be divisors of 100.

We now want to find the maximum number of candies that can be drawn while still leaving ambiguity between the x- and y-flavor scenarios. Let's think of sorting the candies into boxes based on flavor.

- How many boxes can we use? At most x, since otherwise, we'd have more than x flavors.
- How many candies can we put in a box? At most $100/y$, since otherwise, we'd have fewer than y flavors.

Together, this means we can draw at most $x \cdot 100/y$ candies while still being unable to distinguish between the two scenarios. So, to maximize this value, we want to find divisors x and y of 100 (with $x < y$) such that the fraction x/y is as large as possible. Since this fraction increases when y is close to x, we only need to check neighboring divisors of 100.

The divisors of 100 are: 1, 2, 4, 5, 10, 20, 25, 50, 100.
The ratios of adjacent pairs are: 1/2, 1/2, 4/5, 1/2, 1/2, 4/5, 1/2, 1/2.

Among these, the largest is 4/5, and $4 \cdot 100/5 = 80$, which — based on the reasoning above — proves that after 81 candies have been eaten, we can be certain how many different flavors there are.

(Note: since 80 appears twice in this sequence, it means that even with 80 candies eaten, there are still two different scenarios that are indistinguishable. In addition to the already mentioned case of 4 vs. 5 flavors, distinguishing between 20 and 25 flavors also requires 81 candies.)

Remark: In the case of 100 candies, we've seen that more than 80% of the candies must be drawn to be sure. Can you come up with an example where more than 99% need to be drawn? (problem)

Code of the suitcase lock

The fact that the sum of the digits must be 10 will help us. Why is this true? Because the code shows us how many times each digit appears in the code, and there are 10 digits in total.

We will try to find the solution by considering the number of 0s in the code.

We cannot have nine 0s, as then we would require all the remaining digits to be 0, which would mean communicating that we have zero 9s, which is not the case.

0	1	2	3	4	5	6	7	8	9
9									1

If we have eight 0s, we must have one 8 (we cannot have two or more or else the sum would exceed 10), but then the number of 1s is non-zero, so we would have to indicate that and then we would not have enough space for the eight 0s. Also, we couldn't even have 1 as the second digit, as by writing it down we would have two 1s in the code.

0	1	2	3	4	5	6	7	8	9
8	1							1	

Can we have seven 0s? The remaining sum must be 3. We need just one 7 (we cannot have two or more of them), so we need a 1 at the 8th digit (note we are zero-indexed). So, we need something non-zero at the second digit, but we cannot have a 1 here, as by writing that down we would already have two 1s in the code. So we must write 2 for the second digit, as the sum of the digits cannot be larger than 10. So we have a 2, and thus in the third digit we need something non-zero. But our sum is already 10. Also, we would not have enough space for the 0s.

0	1	2	3	4	5	6	7	8	9
7	2	1					1		

Six 0s? Thinking similarly, we can find the code:

0	1	2	3	4	5	6	7	8	9
6	2	1	0	0	0	1	0	0	0

Yeey, we found the code! But wait... Is there another code that works? Try having zero, one, two, three, four, or five for the number of 0s and you'll find that there is no other combination that works for a 10-digit code.

Andi's question:

The number of 0s cannot be 0, because then we would already have one of them. Let's try with only one 0.

We already have a 1, so we cannot write a 1 for the second digit following the same logic as above. The smallest digit we can try is a 2. This means we have to indicate that we have one 2. We can then place a 0 for the 4th digit indicating we have zero 3s (which is true and meets our quota for the number of 0s), and thus we get a 4-digit code: 1210.

Recall that Andi wants the shortest code *and* the smallest number. If we had two or more 0s, we would not be able to get a smaller number than our current code unless it was three or fewer digits. If we had three or more zeros, then just tacking on the indicated zeros would make the number too large (i.e., 3000). So, it could only be 200, which does not work as we have one 2, not zero. It is interesting to note that you can continue working with the example of two 0s to find the valid code 2020 which is also only 4-digits.

While 1210 and 2020 are both correct self-referencing codes of length 4, Andi wanted the smallest such number so the answer to her question is:

0	1	2	3
1	2	1	0

(problem)

What is my number?

Let's take a number *x* and ask if it is close to the number being picked. If the answer is yes, then the secret number must be one of three: x-1, x, or x+1. If we have no further information, we can determine the secret number in two additional questions by asking about $x-1$ and $x+1$.

A natural strategy seems to be taking the numbers 2, 5, and 8. If we get a “yes” to any of these, then, as above, we can determine the number with two more questions. If all three responses are “no,” then the secret number must be 10. Thus, five questions are always enough.

This strategy is elegant and simple. That’s why it’s surprising that fewer questions actually suffice. We recommend that if you haven’t already solved the task with 4 questions, stop reading now and try to improve your strategy.

The improved strategy begins the same way: we first ask about 2, then 5. (If either gets a “yes,” we can finish the task in 3 or 4 questions.)

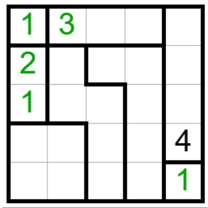
Next, instead of asking about 8, we switch and ask about 7. If we get a “yes” here, then the secret number must be either 7 or 8 (since 6 was already ruled out when we asked about 5). If the answer is “no,” then the number must be either 9 or 10. In both cases, we’re left with two possible numbers, which can be resolved with one final question (asking about 9 or 8, respectively). So, four questions are guaranteed to be enough.

We’ll also reveal this: three questions are not sufficient.

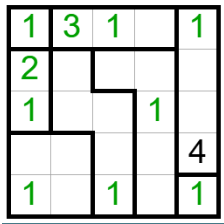
The key difference between the two strategies lies in the third question. When asking about 8, we gain information about three new numbers. When asking about 7, we gain information about only two—but this more focused approach turns out to be more efficient. This example shows that sometimes, being greedy doesn't pay off. (problem)

Non-neighbouring numbers

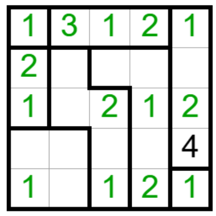
We can write 1s in the two regions of size 1. From this, we can easily fill in the grid to this point:



Consider the two empty squares directly below the remaining empty squares in the region that contains the 3. Neither of these two squares can contain a 1, because one of empty squares in the 3's region must be a 1. And since the bottom-right corner of the grid already has a 1, we know that a 1 must go in the square that shares its bottom-right corner with the upper-left corner of the square containing the 4. After that 1 is placed, the remaining 1s fall into place more easily, starting with the region on the right hand side of the grid.



Now, we can write in some 2s:



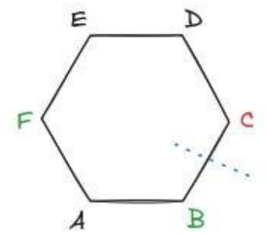
After writing down some 3s, the rest is straightforward:

1	3	1	2	1
2	4	5	4	3
1	3	2	1	2
2	4	5	3	4
1	3	1	2	1

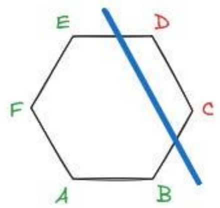
(problem)

On the shores of Themis

Notice that B cannot be a liar, because whether D is telling the truth or lying, based on D's statement, B must be a truth-teller either way. The same applies to F, based on B’s statement. This implies that C must be a liar, so the river must cross the side of the hexagon between B and C.



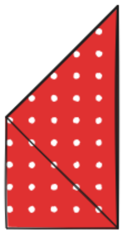
Since the river is straight, A must also be a truth-teller. And A claims that D and E live on opposite sides of the river. Again, using the fact that the river is straight, we can now determine the only possible solution:



Therefore, A, B, E, and F are the truth-tellers. (problem)

Grandpa Tarkal’s scarf

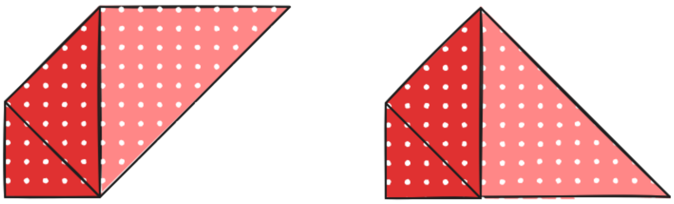
Let’s build Tarkal Grandpa’s quadrilateral out of isosceles right triangles! We begin by placing the first triangle. Then, for each of the next three triangles, we attach one of their legs to the hypotenuse of the previous triangle! There are two ways to do this at each step, but, for the first step, it doesn’t matter which one we choose — the resulting scarf shapes will just be mirror images of each other.



If we continue this process, the triangles will definitely not be the same size, since each one we attach is larger than the last. However, even after placing four triangles, we're not guaranteed to end up with a quadrilateral.

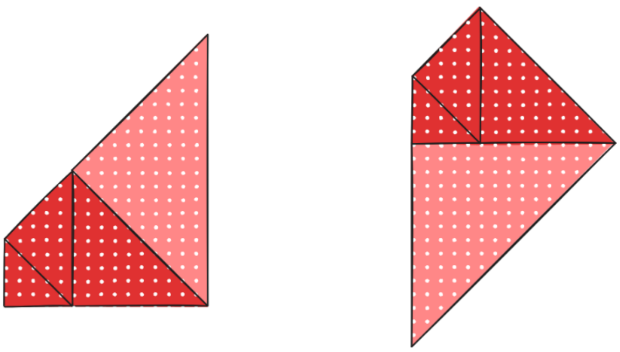
When attaching the **third triangle**, we face a choice: which leg should we glue to the hypotenuse of the second triangle?

Here are the two options:



From here, we only need one more step to complete the shape. The diagram on the right (above) gives us shows the two possible ways to finish the construction:

In one case (shown in the left diagram), the opposite side will have a length of 4 units, while in the other case, it will be 3 units long.



You also found these two examples — *we don't know if there are any more.* (problem)

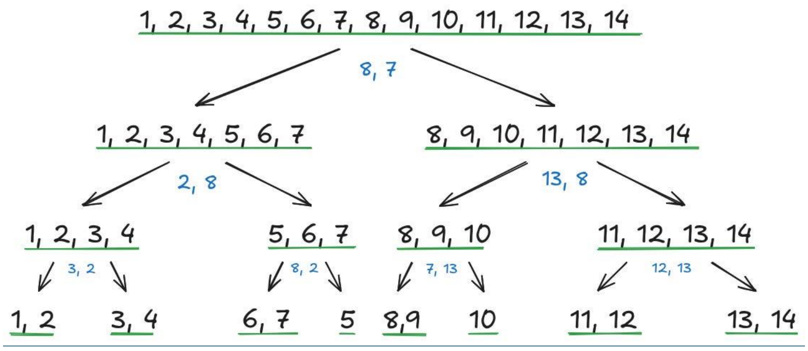
Better and better questions

A natural way to begin is by going through the numbers one by one starting from 1. In this case, you'll get a "closer" answer for a while, and then at some point, it will stop being "closer." The number I picked will be at the turning point. This method takes 14 questions.

We can improve this by going in steps of two or three (and clarifying the uncertain cases around the turning point). This way, it's possible to find the number with just 7 questions.

Before trying to optimize further, let's consider how many guesses we *need at minimum*. In the first round, we don't get any information. From each following round, we receive one of two possible answers. So, the number of possible answer sequences doubles with each new question. That means we need at least 5 (1 + 4) questions, since this allows for 16 different answer sequences—enough to distinguish between 14 possible numbers.

Let's see if we can actually find the number with 5 guesses. Based on the lower bound, it's worth trying to *halve* the potential locations of the target number. For example, we could start with 7 and 8. This approach really can lead to a solution. The details of this (a bit fiddly) process are shown in the following diagram. In the diagram, we used underlining to indicate the possible set of numbers that could have been thought of. The blue numbers show the number we chose in the current and the previous round (we leave the last few questions for the Reader to work out).



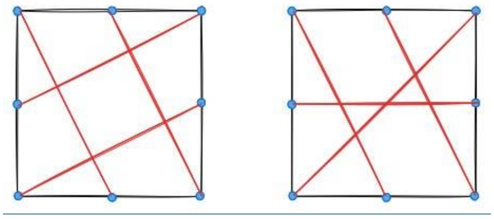
With this, we've solved this problem, since we previously showed that 4 guesses are definitely not enough.

With the specific case worked out, there might still be a lingering feeling of incompleteness. What about the general case? Are 5 guesses enough for 15 numbers too? And for 16? In general, can we determine how far n guesses can take us? We haven't fully thought these through yet—if you feel like it, give it a shot. We don't see a general solution, and it's possible that the answer isn't very elegant.

However, if we modify the rules of the task so that the guessed number can fall *outside* the interval (so, for example, you could guess -2), then there *is* a neat and simple solution. (problem)

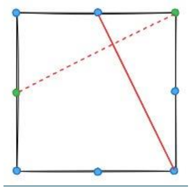
DIY custom picture frame

If we start sketching, after a few attempts we might find constructions with 4, or even 5 intersection points.



Can this be improved? If there are only 2 cords, they can intersect in at most 1 point. With 3 cords, the maximum number of intersection points is 3. If the fourth cord intersects all the previous ones, that adds 3 new intersections. So with 4 cords, the maximum number of intersection points is 6.

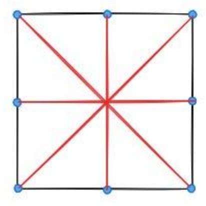
How could 6 intersection points actually be achieved? This is only possible if each cord intersects all the others. Imagine we've already fixed the first cord in place— then, to intersect it, any new cord must have its two endpoints on opposite sides of the first cord.



We need to make the first cord intersect with all three of the others, which means there must be three attachment points on each side of it.

But the same must be true for every cord— so we can only place cords in such a way that each one has three attachment points on either side.

This happens only when each cord connects two opposite points on the frame. If we choose these specific cords, they will all intersect at a single point—the center of the square.



So instead of the desired 6 intersection points, we end up with just 1. Therefore, the construction with 5 intersection points cannot be improved. (problem)

Fraction of fractions

The challenge when trying to find the largest fraction is that we would like to meet a number of conflicting criteria.

When is a fraction large? Exactly when its numerator is large and/or its denominator is small. So we need to find two fractions very close to each other to go in the denominator of the big fraction, while leaving some of the largest numbers for the numerator of the big fraction. However, it is also not good to leave only large numbers in the numerator, because then the quotient of the large numbers would not be really large.

Taking every possible option into account seems exhausting, so let's try a few fractions instead:

$$\frac{\frac{7}{1}+\frac{6}{5}}{\frac{2}{4}-\frac{3}{8}}=65\frac{3}{5}$$

Here, the denominator of the big fraction evaluates to 1/8, but we can easily do better by interchanging the 7 and the 8. Many of you have submitted this solution:

$$\frac{\frac{8}{1}+\frac{7}{2}}{\frac{4}{6}-\frac{3}{5}}=172\frac{1}{2}$$

Here, the denominator is 2/30, which is about half of 1/8. We can increase the value of the fraction by replacing 3/5 in the denominator with 5/7. But this is still not the maximum, we can still find a larger one:

$$\frac{\frac{8}{1}+\frac{6}{2}}{\frac{3}{5}-\frac{4}{7}}=385$$

The value of this fraction is surprisingly large. Is this already the maximum? Could we hope to prove it?

This is indeed the maximum, but the proof is a bit technical. The maximum value of the numerator is: 8/1 + 7/2 = 11.5. If 385 were not the maximum, the following inequality would hold:

$$385 < \frac{\frac{a}{b}+\frac{c}{d}}{\frac{e}{f}-\frac{g}{h}}$$

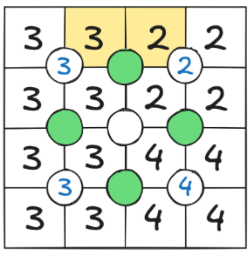
Transforming this inequality, and utilizing our bound on the numerator:

$$\frac{e}{f}-\frac{g}{h} < \frac{\frac{a}{b}+\frac{c}{d}}{385} \le \frac{11\frac{1}{2}}{385} < \frac{1}{33}$$

We have a chance of finding such a difference if the numerator after finding a common denominator is 1, and the *fh* product is at least 33. If the two denominators are 7 and 8, then 1/7-1/8 = 1/56 < 1/33 and 7/8-6/7 = 1/56 < 1/33, but unfortunately in both cases there are numbers that we have used twice. Similarly, 1/6-1/7 = 1/42 < 1/33 and 6/7-5/6 = 1/42 < 1/33 is not correct. If the two denominators are 5 and 8, we need to check the case 2/5 - 3/8 = 1/40; in this case, the value of the fraction is 340, which is less than 385. No other cases are possible, we leave it to the reader to prove this. (problem)

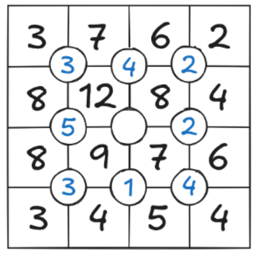
Vault

As the values of the four corner cells can only be increased if the corresponding corner button is pressed, these buttons need to be pressed 3, 2, 4, and 3 times respectively. (On each button we wrote in blue the number of times we have pressed them so far). With the corners now sorted out, we see the following numbers on the lock:



The values of the middle cells in the top row with the yellow background can be increased by only two buttons each: the one in the appropriate corner and the top middle button. As we cannot press the corner buttons any more, we must press the top middle button. This can be done, and to set the values of 7 and 6 we have to press it a total of 4 times.

The same thought process applies to the two middle cells of the bottom row and the two side columns – these can only be set using the corresponding green button too. After performing this, the outer circle of cells contains the correct numbers. We still have to set the values of the 4 middle cells. The following numbers are visible on the lock now:



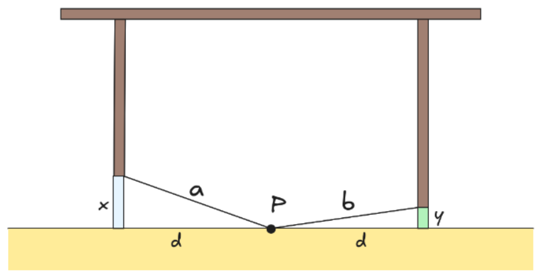
Clearly the only thing we have to do is to press the middle button twice, and with that we manage to open the lock. Note: you only need to figure out 4 of the 9 buttons in order to solve the puzzle. (problem)

Wobbly Table

Let's try to understand how the table wobbles. First, let's add to the animation. What do we see here? The red segment doesn't move at all, because the two longer legs remain on the ground the entire time.

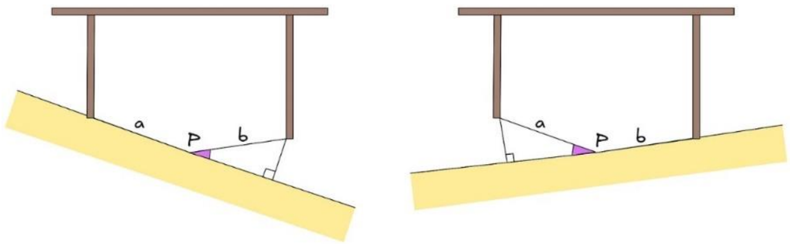
Let the midpoint of the red segment be point *P*. Point *P* and the two shorter legs lie in a single plane, which is also marked in the animation. *P* stays on the ground throughout—it doesn't move—and since it's the midpoint between the longer legs, its position relative to the table's legs also stays fixed. These properties make *P* an important part of our solution.

Now let's look at this plane separately. Imagine we extend the two short legs with additional parts: a blue section (x) and a green section (y) so that all four legs are the same length and the table is level, as shown in the diagram. Assume $x > y$; then $a > b$ also holds, since $a^2 = d^2+x^2$, and $b^2 = d^2 + y^2$.



So what happens in this plane when the table wobbles? For example, if the table tilts to the left, the blue part disappears as side *a* reaches the ground. Similarly, if it tilts to the right, side *b* touches the ground as the green part vanishes.

Let’s analyze these two situations by rotating the diagrams so the tabletop stays horizontal—in other words, instead of the table wobbling, imagine the ground is tilting. In this view:



In both cases, the leg that’s off the ground forms a right triangle, and its height corresponds to the side opposite the purple angle. These two purple angles are equal, because they both complete the angle between *a* and *b* to make a straight line. Therefore, the ratio of the triangle’s sides opposite the purple angles is equal to the ratio of the hypotenuses.

So, the leg with the longer hypotenuse will rise higher. As we saw earlier, $a > b$, meaning the bottom of the shorter leg ends up higher above the ground.

Even though the two heights are different, the difference is visually imperceptible. It can be calculated that the maximum ratio between the two heights is approximately 1.000355. (problem)

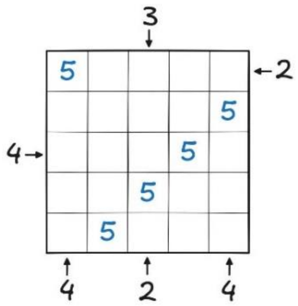
Sketchy map

You can solve the problem by using equations, but let’s think through this in a different way. Imagine that Alice goes from A to D, and Bob goes from B to C. How many kilometers do they go in total? It is obviously 72+54 = 126 km.

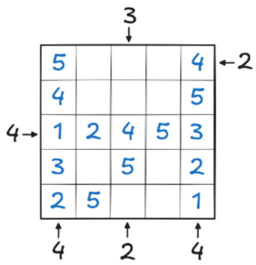
Once they arrive, imagine they go back, but they swap where they end up, so Alice goes to B and Bob goes to A. How many kilometers do they go in total on the return trip? On the one hand, the distance travelled is 50+BD (Bob’s return trip + Alice’s return trip), but on the other hand we observe that the swap does not change the total length, so they have to go a total of 126 km. Thus, 50+BD=126 km, which implies that the length of the route BD is 76 km. (problem)

Housing complex

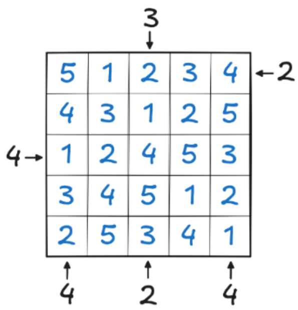
First, let's write the 5-story buildings in the diagram. Because a 5-story building covers all the buildings behind it, the 5s in the two outer columns can only be in the top two rows. A 5 cannot be in the top right corner because there are 2 buildings visible in the top row. So the cell in the top left corner and the rightmost cell of the second row are 5. In the third row, a 5 can only be placed in the 4th column because of the number of buildings visible in that row. The remaining two 5-story buildings can easily be placed using the 2 below the middle column.



For only two buildings to be visible from the right in the top row, the 4-story building must be in the rightmost column. This means that this column is determined, since we also see 4 buildings from the bottom. We can proceed similarly with the third row and the leftmost column.



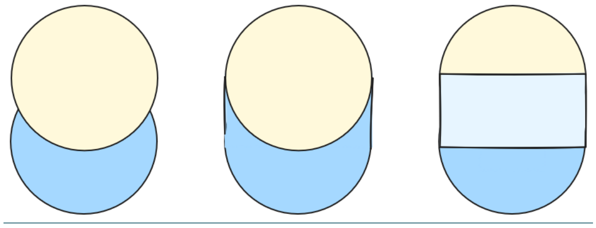
The four empty spots in the lower half of the grid can be filled in based on the rule that each number must appear exactly once in every row and column. After that, the six numbers at the top half of the grid can also be filled in, using the clue that three buildings are visible from the top of the middle column. (problem)



Morning Coffee on the Table

How much easier our situation would be if the side of the mug was straight, not curved! In that case, the water stain would be a rectangle, and we could easily calculate its area.

Unfortunately, it's not that simple - but we'll show that it's not so far off either. Let's draw three diagrams:



In the first diagram, we illustrate the mug's position before (blue circle) and after the movement (yellow circle). Because the mug was dragged across the surface, the water stain is still incomplete. We need to fill in the pathway between the two circles as shown in the second diagram.

Let's think about the area in the second diagram in two different ways. First, we break the shape into three parts (see the third diagram): a top semicircle, a bottom semicircle, and the part in between. The part in between is a rectangle whose one side is the diameter of the mug (8 cm) and the other side is the length of the movement (2 cm). So the area of the shape in the second diagram is equal to the combined area of the two semicircles (which form a full circle) and the rectangle. Second, this shape's area is equal to the mug's circular base plus the water stain. Therefore, the water stain's area outside the mug is exactly the same as the area of the rectangle, which is:

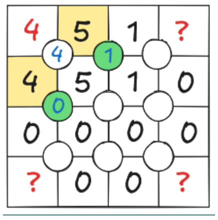
$2 \times 8 = 16\ \text{cm}^2$.

It's no coincidence that a rectangle appears here. It's worth thinking about how this reasoning works for non-circular mugs. If you're up for it, try to determine for which other mug shapes the above reasoning applies. (problem)

Vault with 12 cells

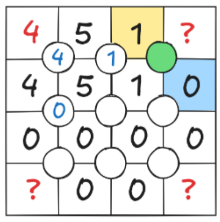
Let's examine the top-left corner! Since this cell's value can only be increased by pressing the top-left button, the total number of times this button is pressed will determine the final value in this cell. Out of the four numbers affected by this button, three are known, with the smallest value being 4. Therefore, we must not press this button more than four times.

Let’s try that first—press the button 4 times! On the figure below, the cells with a yellow background (excluding the corner) are affected by exactly one other button each—the green ones—so those buttons must be pressed either 0 or 1 times. After performing these actions, we are in the following state:

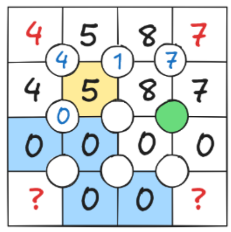


However, we’re not done yet. The cell now marked with a yellow background in the figure below needs to be set to 8, and the only button we can use to reach that is the green one—which would need to be pressed 7 times.

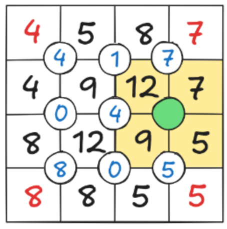
But if we do that, the blue cell will show a 7, even though our target for that cell is 5. So, we definitely won’t reach our goal this way.



Still, let’s ignore this for a moment—often we can learn the most from failed attempts. Based on what we said earlier, press the button in the top-right corner 7 times. After that, we see the following numbers on the lock:



Next, set the desired value on the yellow cell by pressing the center button repeatedly, and then, moving from left to right in the bottom row of buttons, do the same for the blue cells. For now, don’t touch the green button. As a result, we arrive at the following situation:



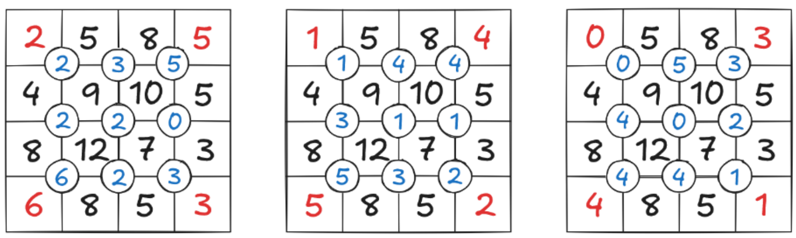
This is interesting because, apart from the yellow cells, all others now show the correct values. Moreover, if we could reduce the yellow cells by 2, everything would be correct. That would mean pressing the green button –2 times. Of course, that's not possible—but the situation can still be improved.

Now start by pressing the top-left button 3 times and proceed by using the buttons in the same order and with the same strategy as above: use the top-middle and left-middle buttons to resolve those cells adjacent to the cell in the top-left corner, then use the top-right button, the middle button, and lastly the bottom row of buttons from left to right.

We arrive at a very similar situation: the green button used for adjusting the yellow cells now only needs to be pressed –1 times.

If we reduce the top-left corner value further, we'll find a correct solution, since 0 is a valid value for the green button. From this point, it's easy to see that the top-left corner can be set to 2, 1, or even 0—and the lock can be opened in three different ways. (In these cases, the center button would need to be pressed 2 times, once, or not at all.)

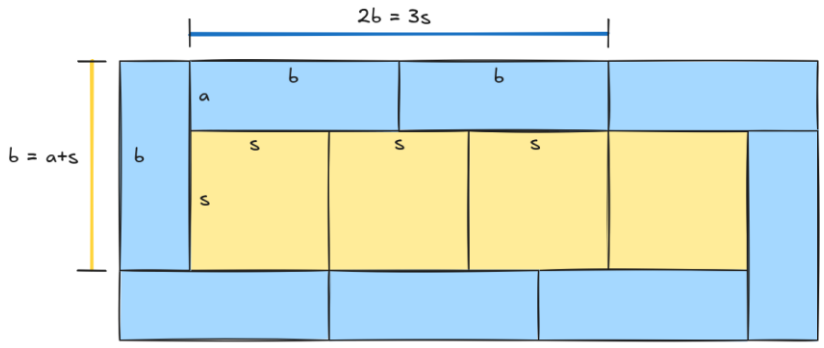
The figure below shows all three valid solutions:



(problem)

Mosaic floor

Let the sides of the blue rectangle be *a* and *b*, and the side of the square be *s*. Let's take a look at the diagram:



Let's find relationships between the sides. The segment marked in yellow is, on one hand, the longer side of the blue rectangle, and on the other hand, it is the sum of the shorter side and the side of the square, so $b = s + a$.

We can also express the blue-marked segment in two ways, which gives us: $2b = 3s$.

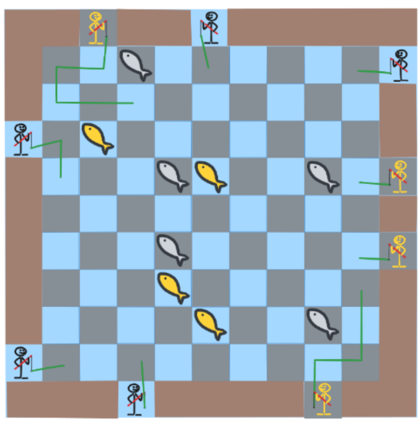
Doubling the first equation gives $2b = 2s + 2a$. Comparing this with the previous equation, the two right sides must be equal: $2s + 2a = 3s$. So, the side of the square is twice the shorter side of the rectangle. The longer side of the rectangle is three times the shorter side, because $b = s + a = 3a$.

Now let's write the perimeter of the large rectangle in terms of a, the shorter side of the blue rectangle: $P = 8b + 4a = 28a = 42$, meaning $a = 3/2$.

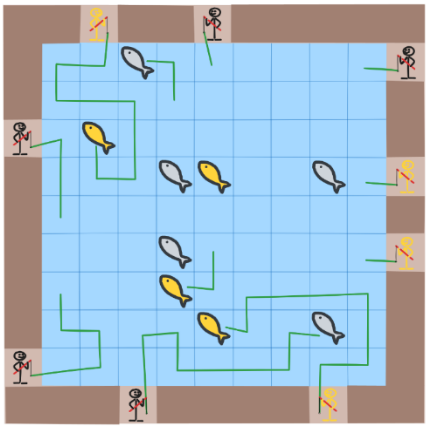
Now we can find the area: $A = (b + a) \times (3b + a) = (3a + a) \times (9a + a) = 4a \times 10a = 6 \times 15 = 90$ m^2.
(problem)

Fish in a lake

If we color the tiles using a checkerboard pattern, then the lines will alternate between passing through dark and light tiles. This means a 9 tile long fishing line, assuming it comes from an angler standing on a dark tile, will end up at a fish on a light tile, and vice versa. Based on this, the yellow fish will correspond to the yellow anglers.

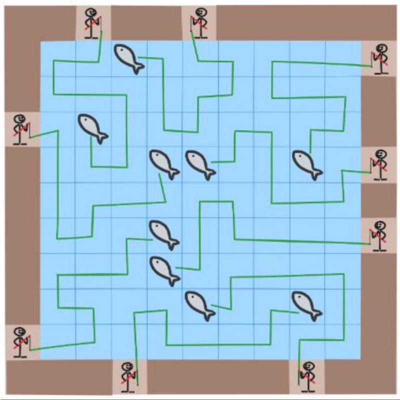


This way it can be seen that the bottom right fish can only belong to the left angler on the bottom edge, while the fish in the second column can only be reached by the line of the top left angler.



As the combined length of the fishing lines is the same as the number of tiles on the board, we know that each tile must have a line passing through it.

The top rightmost fish can only belong to the top right angler. In order for her line to reach this tile, the line of the middle angler of the right edge must go around this fish from below. From here, the rest of the board can be filled out in a few steps:

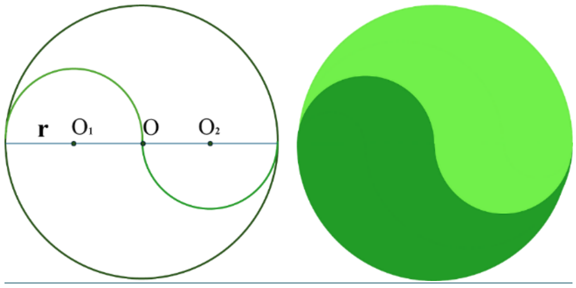


(problem)

Island of different tea types

Draw a diameter of the circle, and then draw two semicircles around two quarter points of the diameter, in opposite directions, as shown in the left figure!

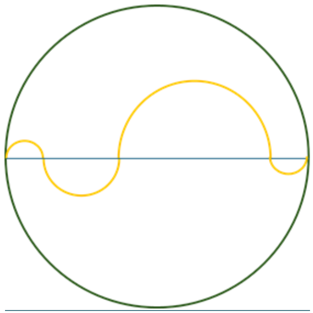
This way, you get a good division — and actually, a well-known symbol.



Why is this division good? The two parts have equal areas because they are rotations of each other. If the radius of the large circle is 2·*r*, then its circumference is 4·*r*·π. The arc length of one small semicircle is *r*·π. The perimeter of one tea field is the sum of one large semicircle and two small semicircles, that is: 2·*r*·π + *r*·π + *r*·π = 4·*r*·π, which indeed matches the island's circumference.

Just in case you're interested 🙂 — if not, feel free to skip this part.

As a curiosity, we mention that no matter how you "wave" the diameter of a circle using semicircles, the length of the wavy line will always equal half of the full circle's circumference.



This is because the sum of the radii of the small circles equals the radius of the large circle. In detail:

$$r_1\pi + r_2\pi + \cdots + r_n\pi = (r_1 + r_2 + \cdots + r_n)\pi = R\pi$$

where *R* is the radius of the full circle, and r_1, ..., r_n are the radii of the small circles.

Those who are a little more familiar with the world of curves can draw many other constructions — either based on the previous "waving" method or using different ideas. Moreover, it is even possible to give a solution to the problem that only proves the existence of such a division, without explicitly constructing it. (problem)

With friends like these…!

First, let us determine the maximum number of truth-tellers in the group.

Denote by k the number of truth-tellers. Truth-tellers all tell the truth, so both of their friends in the group are indeed liars. Therefore, if we go over all the truth-tellers and count their friends, then we count at least $2 \cdot k$ liars. However, this way a liar could have been counted multiple times, for example when the given liar has multiple truth-teller friends. As everyone, including liars, has only two friends, every liar must have been counted at most twice. Consequently, there are at least $(2 \cdot k)/2 = k$ liars in the group. In other words, there are at least as many liars in the group as truth-tellers.

This way we know there are at most 6 truth-tellers, which can actually be realized: have 12 people sit around a table such that truth-tellers and liars alternate. If everyone is friends with their two neighbors, then the assumptions of the problem are satisfied with 6 truth-tellers among the people.

Let us now determine the minimum number of truth-tellers in the group.

We previously counted the liars grouped with respect to the truth-tellers; let us now work the other way around. Every liar makes the claim "Both my friends are liars." This is a lie, so each liar must have a truth-teller friend.

Let h denote the number of liars. Then while going over the friends of liars we count at least h truth-tellers, as each liar has at least one such friend. It can happen that a truth-teller gets counted multiple times this way, for example when a truth-teller has multiple liar friends. However, it is guaranteed that each truth-teller was counted at most twice, because that is the number of friends everyone has. Therefore, there are at least $h/2$ truth-tellers in the group. As there are at least half as many truth-tellers in the group as liars, at least 4 of the 12 people must be truth-tellers.

Such a group with only 4 truth-tellers actually exists: have 12 people sit around a table following the pattern LTLLTLLTLLTL, where each L denotes a liar and each T a truth-teller. If everyone is friends with their two neighbors, then the assumptions of the problem are satisfied with 4 truth-tellers among the people. (problem)

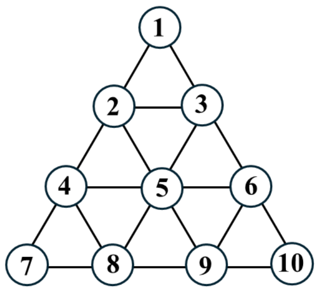
A stream, rails, and a trail

If we cross the stream once, then we get to its other side; if we cross twice, then we get back to the original side. Since after every second crossing we get back to the original side, if we cross eight times, then we end up on the original side. As such, at the end of the hike the stream will once again be to the left of us.

We cross the rails five times, ending up on its other side, so at the end of the hike it will be to the right of us. Consequently, the order of the three at the end of the hike is: stream, trail, rails. (problem)

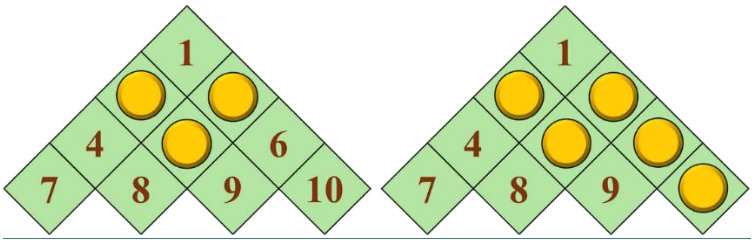
Let’s jump!

Eight tokens can be removed with five steps. To reach our goal, we can examine the structure of the board, which is actually rotationally symmetric. At first glance this might not be obvious, but if we consider the layout of the squares and the possible moves, it becomes clear that it corresponds nicely to a triangular grid like this:



But let's get back to the board!

It’s worth thinking about how we can achieve longer moves. The 2-3-5 configuration looks promising because all three tokens can be removed in a single step. Starting from 6, we end up back at the same spot. If we also have a token on square 10, then we’ll be able to remove the one that lands on 6 with another step. (Looking at the board differently, an equivalent setup can be represented by placing tokens on 5-6-7-8-9 or 4-5-8-9-10 for example.)



Based on the previous ideas, here’s a possible five-step solution: Leave square 2 empty when placing the tokens!

7 → 2

1 → 4

9 → 7 → 2

6 → 1 → 4 → 6

10 → 3

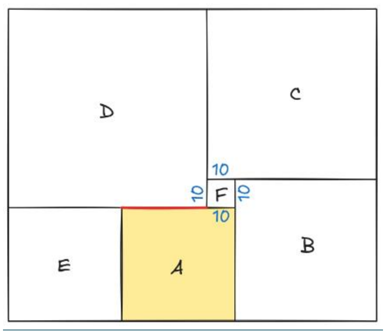
Due to the board’s structure, instead of leaving square 2 empty at the start, you could have left 3, 4, 6, 8, or 9 empty. Each of these squares lies along the sides of the triangular board, not at the vertices or in the center. Our moves can easily be adapted to these alternative starting positions.

It’s clear that if we leave the center square (5) empty at the beginning, no token can move at all.

Also, if we leave a corner square empty, we won't be able to achieve our goal either. This can be fairly easily tracked: during the game there will be forced moves (if we consider the rotational and reflective symmetries), and there will be branching points where multiple moves are possible. Going through these, we can see that on none of the branches can we remove eight tokens. (problem)

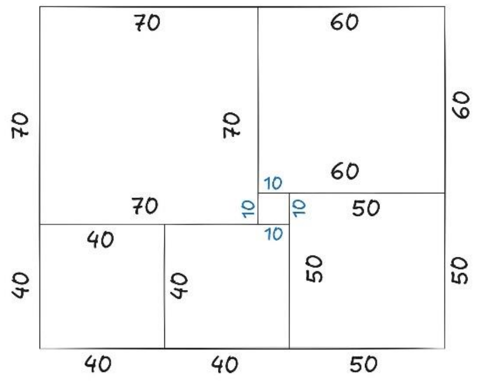
Brickville

Since the castle used to consist of 6 square-shaped rooms, with 1 fully visible and 5 partially visible in the excavation sketch, there is no room that is not at least partially shown in the diagram. Based on this, we can complete the floor plan of the castle as shown in the sketch below.



Rooms A and E are the same size, since they share a common wall. Starting from room A (yellow) and going counterclockwise, let’s walk around the smallest room, F. Room B is 10 feet longer on one side than room A, due to a partially shared vertical wall. Similarly, room C is 10 feet longer than room B (due to a partially shared horizontal wall). Room D is another 10 feet longer than that. So, room D's side is 30 feet longer than the bottom left room, E. This extra length is shown by the red section in the diagram above.

Therefore, room A's side is 30 + 10 = 40 feet, and the other rooms measure 50, 60, 70 feet, respectively, as shown in the next diagram.



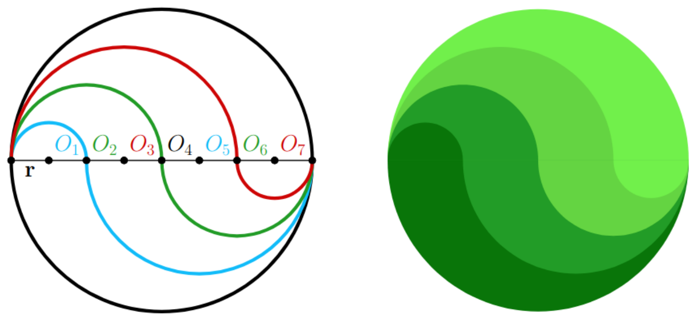
So, the length of the castle's shorter side is 110 feet. [(problem)](#)

Island of different tea types 2

We received many diverse and creative drawings — you can browse through all the submitted diagrams here. Now, we'll show you a partition that's different from these.

Divide the diameter of the island's circle into eight equal parts, and label the division points as O_1, O_2, O_3,..., O_7!

Then, using these points as centers, draw semicircles with radii r, $2r$, and $3r$ according to the diagram! The colors indicate which arcs belong to which centers. The circle with radius $4r$ is the original circle of the island.



If you hadn't thought of this solution, now might be a good moment to have a closer look at why it works well.

1. Are the perimeters of the four tea fields identical?

In the solution to the first version of *Island with different tea types*, we showed as a curiosity that no matter how we 'wave' the diameter of a circle with semicircles, the length of the resulting wavy line always equals half the circumference of the original circle.

We can now refer back to that idea, but we’ll also demonstrate it directly for this case: The length of the blue "wave" equals the sum of the small upper semicircle and the large lower semicircle, that is: $r \cdot \pi + 3r \cdot \pi = 4r \cdot \pi$, which is exactly half the circumference of the large circle (the island).

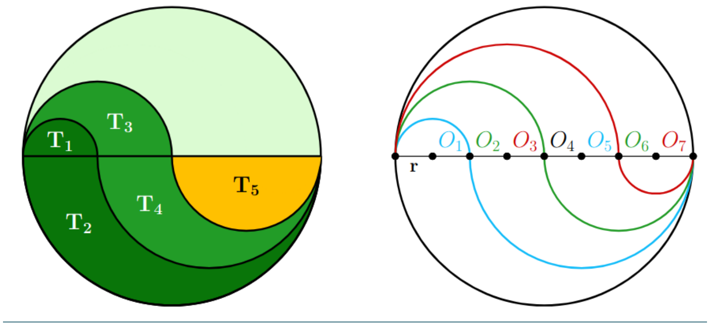
The green "wave" consists of two semicircular arcs with radius 2r each, and its length is $2r \cdot \pi + 2r \cdot \pi = 4r \cdot \pi$, again half the circumference of the large circle.

The red "wave" is made up of arcs identical to the blue ones, so it too equals half the circumference of the large (island) circle.

Thus, the perimeter of each tea field equals the full circumference of the island.

2. Are the areas of the four tea fields identical?

Let’s examine the two darker green areas — the other two tea fields are congruent to one of these.



The area of the dark green tea field is: $T_1 + T_2$, the area of the light green tea field is: $T_3 + T_4$.

T_1 and T_5 are semicircles. Each of the more interesting-shaped regions can be obtained as the difference of the areas of two semicircles.

For example, T_3 can be calculated by subtracting T_1 from the corresponding larger semicircle. We can apply the same logic for the areas below.

In more detail: Keep in mind that if we change the radius of a circle by a factor of *n*, its area changes by a factor of n^2:

$(nr)^2 \cdot \pi = n^2 \cdot (r^2 \cdot \pi)$. The same is true for halves of circles as well.

$T_3 = 4T_1 - T_1 = 3T_1$

$T_5 = 4T_1$

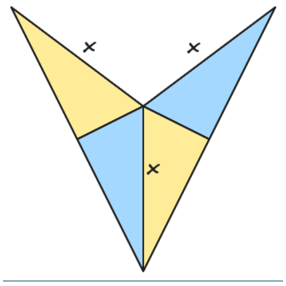
$T_4 = 9T_1 – T_5 = 9T_1 – 4T_1 = 5T_1$

$T_2 = 16T_1 - 9T_1 = 7T_1$

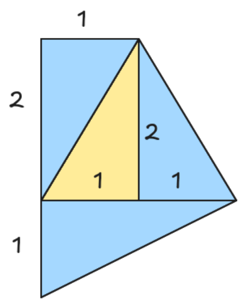
Thus $T_1 + T_2$ and $T_3 + T_4$ are exactly $8T_1$. That is, we got a proper partition. (problem)

Non-trapezoid

We can construct a concave quadrilateral from two identical isosceles triangles, and each of those can be divided into two congruent right triangles. That gives us the following example:



If we stick to right triangles, we can also choose the legs in such a way that we get a convex quadrilateral:

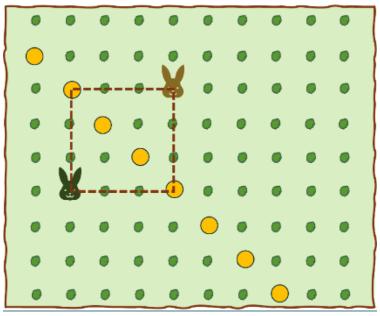


We haven't compiled the figures you sent this time. These two were popular among you too, and several of you also shared examples starting from an equilateral triangle — some even involved other types of deltoids. (problem)

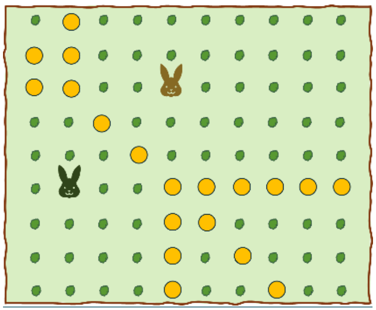
Cabbage field

The answer is 32. If your result is different, it might be worth pausing and thinking a bit more at this point.

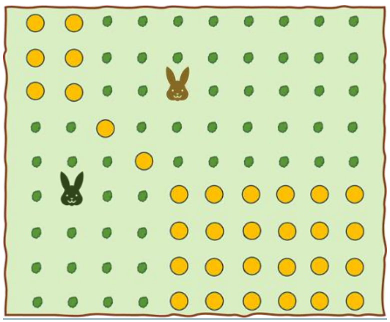
After a little pondering, the four cabbages found on the diagonal of the square with opposite corners on the rabbit holes are easily seen to be marked for donation. Even more, we may extend the diagonal in both directions, because this way we move away from the two rabbit holes by equal amounts.



Now consider the cabbage growing in the corner of the square between the two rabbits. From this cabbage, could we continue jumping so that we distance ourselves from the two rabbit holes equally? Perhaps surprisingly, yes!



Even more, following this train of thought we can find further cabbages in the top left and bottom right rectangles whose sides we already marked in the figure above:

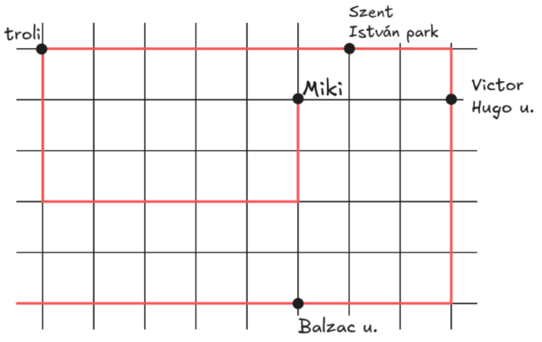


This way 32 cabbages are donated by the rabbits.

An interesting concept of “distance” appears in this problem. In our math programs we explore some strange situations arising from it and discover its sometimes really surprising properties. (problem)

Trolleybus 76

Let’s call the distance between two intersections one block. Note that the stop Miki is currently at is 2 blocks away from the Szent István Park stop and 3 blocks away from the Victor Hugo Street stop. If Miki heads toward Victor Hugo Street, he will be 1 block away from the Victor Hugo stop when the trolleybus arrives at Szent István Park. This means that while the trolley travels from Szent István Park to Victor Hugo Street (a distance of 3 blocks), Miki only covers 1 block to reach the stop. Thus, the trolley moves three times as fast as Miki. Since Miki is currently 2 blocks away from Szent István Park, the trolley must be 6 blocks away from there in order to arrive at the same time as Miki does. We can now put the trolleybus on the map:



We see that the trolleybus must have traveled 10 blocks in the 5 minutes prior to Miki's arrival to his current location, which means it covers 2 blocks per minute.

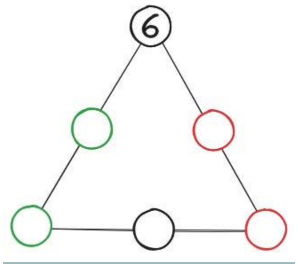
Miki is 4 blocks away from the Balzac Street stop. By the time he walks 3 blocks toward the stop, the trolley reaches the Victor Hugo Street stop. From there, Miki goes 1 more block, while the trolley travels 3 more blocks. So when Miki arrives at the stop, the trolley is still 4 blocks away from Balzac Street.

Since the trolley moves at a speed of 2 blocks per minute, it will cover the remaining 4 blocks in 2 minutes and that's how long Miki would have to wait if he chooses the Balzac Street stop. (problem)

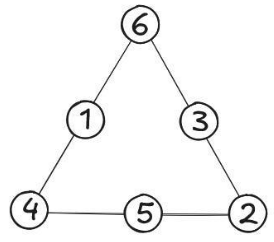
Pebbles in a triangle

It's worth messing around to come up with an answer—an appropriate arrangement can often be found intuitively. Here, we'll show a solution that reaches the answer in a more targeted way.

Let's pick one number, for example, 6. Let's see if it's possible for 6 to be at one of the triangle's vertices. The sum of the numbers on the two adjacent sides must be equal, which means that the numbers placed in the green and red circles in the figure below must sum to the same value.



So we need to select two pairs from the remaining numbers (currently 1, 2, 3, 4, and 5) such that both pairs have the same sum, and the leftover number goes to the midpoint of the bottom side. In our example, we can choose 1+4 = 2+3, which leaves 5 to be placed at the bottom. Since the sum along one side is now 6+1+4 = 11, we need to place the numbers into the red and green circles in such a way that the horizontal bottom side also sums to 11, including the 5. This can easily be done as shown below.



You can also try starting with a number other than 6, or explore whether the remaining numbers can be grouped into different pairs with equal sums. In this way, three more solutions can be created, making four in total. We leave the discovery of these to the reader. (We consider two solutions the same if one can be transformed into the other through reflection or rotation.) (problem)

Dystopia

Let's choose an arbitrary pair and ask A if B is an artificial intelligence.

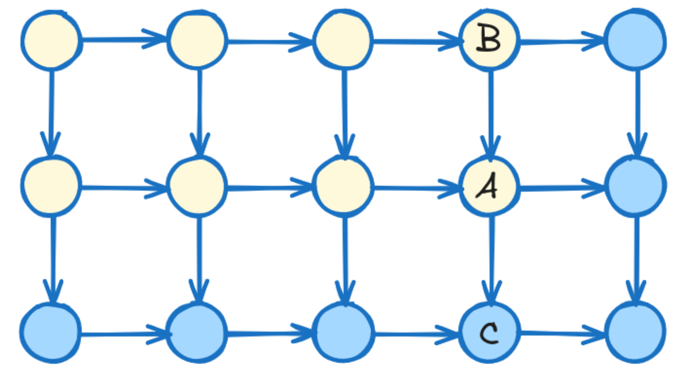
- Case 1: A says that B is an artificial intelligence. Since A is truthful, B indeed is an AI, so we can eliminate B as a candidate for being the human. Then, we ask A about another user who hasn't yet been checked.
- Case 2: A says they don't know if B is an artificial intelligence. Since the human knows for certain that everyone else is an AI, A cannot be the human, so we can eliminate A as a candidate. Then, we ask B about another user who hasn't yet been checked.

With this strategy, in each step we can eliminate one previously unchecked user from being human. Since there are 13 users in total, after 12 questions we will definitely know that the only

user who has not been eliminated is the human. It’s not possible to find the human with fewer than 12 questions.
Imagine that each time we ask, the answer is always that the other user is an AI. In this case, we can only eliminate the one being asked about, but we learn nothing about the one who answers. So, with each question we can eliminate at most one candidate — no matter how we ask. Here’s an alternative proof. Let's suppose we arrange the users in order of intelligence, with the human being the most intelligent. Furthermore, assume that a more intelligent user knows if a less intelligent user is an artificial intelligence, whereas a less intelligent user does not know if a more intelligent user is an AI. This way, the task becomes easier than in the original problem, but we will prove that even so, 12 questions are still necessary. The question we ask of A about whether B is an AI tells us which one is more intelligent. We can represent the users as the vertices of a graph, where there is an edge between two vertices (users) if we have asked a question about that pair. After 11 such questions, the graph cannot be connected (13 vertices and only 11 edges), meaning there will be at least two sets of vertices between which there is no edge. Consequently, we cannot determine which of these two groups has the more intelligent user. Therefore, 11 questions are insufficient to definitively identify the most intelligent user, the human. (problem)

Contamination

With 4 tests, we can identify the source of the contamination. Let our first test be at the pool labeled A on the diagram.

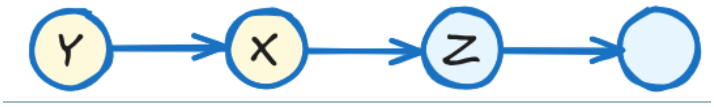


If A is contaminated, then the source must be one of the pools marked in yellow. In this case, our next test should be at pool B. If B is contaminated, then the source is in the first row. If B is not contaminated, the source is in the second row.

If A is not contaminated, then the source of contamination must be among those marked in blue. We then test pool C. If C is contaminated, the source lies among the first four pools in the third row. If C is not contaminated, the source is in the fifth column.

In all four possible scenarios, we have narrowed down the source of contamination to a chain of 3 or 4 pools. In such a chain, we can always find the contamination source with two additional tests.

Let's take a look at this diagram:



We first test pool X, then, depending on the result, we test either Y or Z. If X is contaminated, we still need to check Y to see if it is the source or not. If X is not contaminated and the chain is only 3 pools long, then Z is the source. Otherwise, we check Z. If Z is contaminated, then it is the source, but if it is also not contaminated, then we know the fourth in the chain is the source (this situation corresponds to when we confirmed that the row containing A, B, or C was contaminated in the above diagram).

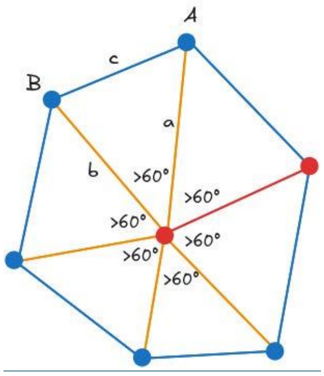
So two additional tests are indeed sufficient meaning that 4 in total will determine which pool is the source. It can be reasoned that three tests are not enough, but we leave that reasoning to the reader. (problem)

Picture book

The first animal was a pigeon. Before the second sound it's possible that the child saw a dog or a goose, but there could have been at most one of each animal — because if there had been more of any one kind, we would have heard its sound. So, the second imitation happens at the second pigeon. Before the third imitation, there could have been at most two of each animal — because if there had been more of any one kind, we would have heard its sound. Since the third imitation is a goose (honk-honk), we know that the child has already seen 2 geese and that the third sound occurs at the third goose. Let's think about what happens when the child makes a sound. It always means that the animal they just saw has now appeared more times than either of the other two. So each time a sound is made, that animal takes the lead (or increases their lead) by one. For example, the seventh imitation is a woof-woof, which means the child is now seeing their seventh dog and the number of pigeons and geese is less than that. Since the last (ninth) imitation is honk-honk, that means the number of geese is exactly 9. (problem)

Soccer field

There will definitely be at least two players with balls at the end, because the pair of people which span the shortest distance are guaranteed to pass to one another. Suppose they end up being the only people left with balls. Then one of them would have at least 6 balls, meaning that player is the closest neighbor of 6 other players.



We illustrated this situation in the figure, with the player with 6 balls in the center. Let us take a look at the triangle marked with letters. If side *a* was greater than side *c*, then person A would have to pass to player B instead of the center player. Similarly, $c > b$. Therefore, the longest side of the triangle is *c*, meaning the angle opposite to it is the largest, and is definitely larger than 60°.

The same reasoning applies to the other triangles. But this is impossible, because then the sum of the angles around the center point would be greater than 360°. As such, it is impossible that only two players get balls.

While this argument was a little technical, what we actually showed was that the players cannot sit too densely around the players who are left with the balls at the end.

We can position 5 players around one player so that everyone passes to the one in the center. Two such arrangements can also be combined. This way, it is possible to have three players left with balls, and here is an example: